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Masses of Flavor Singlet Hybrid Baryons

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Abstract

We study the possibility that four iso-singlet baryons $\Lambda_s(1405)$ $J^P = \frac{1}{2}^{(-)}$, $\Lambda_s(1520)$ $J^P = \frac{3}{2}^{(-)}$, $\Lambda_c(2593)$ $J^P = \frac{1}{2}^{(-)}$ and $\Lambda_c(2625)$ $J^P = \frac{3}{2}^{(-)}$ are hybrids: three quark one gluon states ($udsg$). We calculate the mass separations of the candidates, using a degeneracy-lifting hyperfine interaction from an effective single colored gluon exchange between the constituents. The correct ordering of masses is obtained (contrary to the case for the conventional interpretation as 3 quarks with $L = 1$) and the splittings are plausible. The parity of these states is not measured, only assumed to be negative. In the hybrid picture, the lightest states are parity even and the parity odd counterparts lie about 300 MeV higher. Thus the hybrid ansatz predicts that either the parity of the $\Lambda(1405)$ etc is positive, or that there are undiscovered positive parity states about 300 MeV lower. We also remark that in this picture, the H-dibaryon mass may be around 1.5 GeV.

1 Introduction

The spectrum of hadrons between 1 GeV and 2 GeV has been accurately measured experimentally [1]. In the '60s, Gell-Mann and Zweig [2] proposed a systematic classification of the hadrons into multiplets based on quarks, the fundamental building blocks of matter. The fact that free quarks had never been observed led Gell-Mann at that time to argue that quarks are just a mnemonic device to explain the observed spectra. The understanding of non-observation of quarks was interpreted by *Quantum Chromodynamics* (QCD) in the '70s. It is now generally assumed that QCD is the fundamental theory of the strong interactions. The quarks that build up the hadrons are bound together by virtual gluons, the mediators of the strong force. QCD is an asymptotically free theory; at short distances (i.e. large momentum transfer) the quark-gluon coupling goes to zero and conversely, when quarks are separated from each other by large distances, the coupling gets stronger. This effect, known as "infrared slavery", leads to quark confinement at low energies. Matter consists of color singlets, which are objects with zero total color charge such as mesons ($q\bar{q}$) or baryons (qqq).

QCD predicts other composite particles, color singlet states not only involving quarks but also "constituent" gluons [3]. The constituent gluons should not be confused with the virtual gluons. The constituent gluons contribute with their spin and parity to the quantum numbers of the hadron. The phenomenology of these baryons, called hybrids, was developed by Barnes and Close [4, 5], Golowich et al. [6] and others. These works use the bag model and the potential model to suggest that the lightest hybrid masses are below 2 GeV. Although there is evidence in favor of some candidates [7], the hybrids have not yet been found. One problem is that the mixing of hybrids with conventional states makes them hard to detect. Since the quantum numbers of hybrid states sometimes coincide with those of known particles, it is also possible that some particles identified as conventional baryon states could be hybrids [8, 9].

It would be interesting if the spin $\frac{1}{2}$ and $\frac{3}{2}$ isosinglet baryons conventionally identified as an $L = 1$ state in the quark model turned out to be a hybrid baryon state. A constituent gluon in a negative parity mode combining with quarks in an $L = 0$ orbital ground state can give $J^P = \frac{1}{2}^{(-)}$ and $\frac{3}{2}^{(-)}$ states. The $\Lambda(1405)$ $J^P = 1/2^{(-)}$, which is still a mystery in the quark model, could be one such candidate [11]. Composed of an up, a down and a strange quark, the $\Lambda(1405)$ is usually assumed to be a state with orbital excitation [12]. However there is a major problem in this quark model interpretation, because it implies the $\Lambda(1405)$ and its partner, the $\Lambda(1520)$ with $J^P = 3/2^{(-)}$, are predicted to be nearly degenerate in mass [13]. Even worse, calculations with spin orbit interactions [14] predict an inversion of the masses. As a second interpretation, some suggest that the $\Lambda(1405)$ is a bound state of $\bar{K}$ and N (or a resonance of a pion and a Σ [15]). The problem with this interpretation is that it only provides a $J^P = 1/2^{(-)}$ state and not a $J^P = 3/2^{(-)}$ state. If the $\Lambda(1520)$ is given the conventional interpretation as an orbital excitation of 3 quarks, another particle with $J^P = 1/2^{(-)}$ [16] whose mass is close to the $\Lambda(1520)$ is required. This is ruled out experimentally since this region has already been explored thoroughly.

In this paper we investigate the possibility that the $\Lambda(1405)$ and its partner $\Lambda(1520)$ are hybrids. To test this hypothesis, we calculate the mass splittings of the flavor singlet, octet and decuplet hybrid baryons. In section 2 we derive the general structure of the hybrid wave function. In section 3 we introduce the degeneracy lifting hyperfine interaction. The interaction strength is determined by an effective 1 gluon exchange between quarks and

between a quark and a gluon. In section 3.3, the strength of the quark-quark effective hyperfine coupling is determined from ordinary baryon mass splittings. In section 4 we calculate the flavor singlet hybrid mass splittings and discover they have the correct ordering $m_{3/2} > m_{1/2}$, unlike in the conventional orbital excitation picture. We then use the observed flavor singlet splittings to fix the quark-gluon effective hyperfine coupling. This allows predictions to be made for octet and decuplet hybrid baryon mass splittings. In sections 5 and 6 two immplications of this model are discussed – parity doubling and a light H-dibaryon. Section 7 gives a summary of our results and conclusions.

2 Wave functions of hybrids

In this section we study the structure of the hybrid wave function. It is determined by the three quark and the constituent gluon wave functions. We discuss wave functions for quarks in orbital ground states. Because the quarks have to obey the Pauli principle, we can show that a systematic group theoretical classification of all hybrid wave functions is possible.

2.1 Construction of the quark wave functions

The $SU(N)$ group for color is $SU(3)_C$. A quark transforms like a triplet under $SU(3)_C$ because it comes in three colors. The $SU(N)$ group for spin is $SU(2)_S$. The spin of the quark is $1/2$, so it is a doublet representation of $SU(2)_S$. In this paper we consider systems of three quarks having up to three different quark flavors. The $SU(N)$ group for flavor is then $SU(3)_F$, if the mass differences between quarks of different flavors are neglected. A quark transforms as a triplet under $SU(3)_F$, if flavor symmetry is assumed. A single quark is then a 18-dimensional representation of the direct product group $SU(3)_C \times SU(3)_F \times SU(2)_S$. We reduce the direct product of the three quarks in irreducible representations of $SU(18)$; with the help of Young tableaux [17] we find:

$$\underset{\mathbf{18}}{\square} \times \underset{\mathbf{18}}{\square} \times \underset{\mathbf{18}}{\square} = \underset{\mathbf{1140}}{\square\square\square} + \underset{\mathbf{1938}}{\begin{array}{l}\square\square\\ \square\end{array}} + \underset{\mathbf{1938}}{\begin{array}{l}\square\square\\ \square\end{array}} + \underset{\mathbf{816}}{\begin{array}{c}\square\\ \square\\ \square\end{array}} . \tag{1}$$

We label the resulting irreducible representations by their dimensions. The completely antisymmetric representation is the **816**. All qqq-states that obey the Pauli principle are members of this multiplet. We begin to decompose the multiplet into representations of $SU(3)_C$, $SU(3)_F$ and $SU(2)_S$. We find eight different representations of flavor, color and spin (see table 1). We can check if we have listed all possible decompositions. By counting the numbers of the states in table 1 we find 816, so we are consistent.

The gluon is in the color octet representation of $SU(3)_C$. The hybrid has to be a color singlet. Therefore, the quarks have to be in the complex conjugate representation. This is again an octet, so only color octet qqq-states are of interest here. We can discard all other states. We will distinguish the color octets in the lower half of table 1 by the short hand ${}^{spin}flavor$: ${}^{\mathbf{2}}\mathbf{8}$, ${}^{\mathbf{4}}\mathbf{8}$, ${}^{\mathbf{2}}\mathbf{10}$, ${}^{\mathbf{2}}\mathbf{1}$. There are altogether 70 color octet states forming a $SU(6)$ representation of $SU(3)_F \times SU(2)_S$ with mixed symmetry:

$$\underset{\mathbf{6}}{\square} \times \underset{\mathbf{6}}{\square} \times \underset{\mathbf{6}}{\square} = \underset{\mathbf{56}}{\square\square\square} + \underset{\mathbf{70}}{\begin{array}{l}\square\square\\ \square\end{array}} + \underset{\mathbf{70}}{\begin{array}{l}\square\square\\ \square\end{array}} + \underset{\mathbf{20}}{\begin{array}{c}\square\\ \square\\ \square\end{array}} . \tag{2}$$

$SU(18)$	$SU(3)_C$	$SU(3)_F$	$SU(2)_S$	# of these
$\mathbf{816}$	$\mathbf{1}$	$\mathbf{8}$	$\mathbf{2}$	16
	$\mathbf{1}$	$\mathbf{10}$	$\mathbf{4}$	40
	$\mathbf{10}$	$\mathbf{8}$	$\mathbf{2}$	160
	$\mathbf{10}$	$\mathbf{1}$	$\mathbf{4}$	40
	$\mathbf{8}$	$\mathbf{8}$	$\mathbf{2}$	128
	$\mathbf{8}$	$\mathbf{8}$	$\mathbf{4}$	256
	$\mathbf{8}$	$\mathbf{10}$	$\mathbf{2}$	160
	$\mathbf{8}$	$\mathbf{1}$	$\mathbf{2}$	16

Table 1: Decomposition of $SU(18)$ in representations of color, flavor and spin.

Young tableaux of the form are called mixed symmetric because the wave function which is assigned to the tableaux is neither completely symmetric nor antisymmetric. But the wave function still has defined symmetry properties under the exchange of two quarks. We choose them to be the first and second quark and will use this convention throughout. In the wave function the first and second quark can be either coupled in a symmetric or antisymmetric way, shown below.

$$\begin{array}{rcl} SU(3): \ \mathbf{3}\times\mathbf{3} &=& \mathbf{6}+\bar{\mathbf{3}} \\ SU(2): \ \mathbf{2}\times\mathbf{2} &=& \mathbf{3}+\mathbf{1} \end{array} \qquad (3)$$

We label the wave function φ by a subscript MS for mixed symmetric states (φ_{MS}) and MA for mixed antisymmetric states (φ_{MA}) to indicate these important symmetry properties of the first and second quark. Because the choice of this basis is arbitrary, we give the resulting form of the wave function explicitly in appendix C. We can build a totally symmetric (totally antisymmetric) function $\phi_{S(A)}$ out of two mixed symmetric functions $\varphi_{MS(A)}$ and $\varphi'_{MS(A)}$ in the following way [18]:

$$\phi_S = \frac{1}{\sqrt{2}}(\varphi_{MS}\varphi'_{MS} + \varphi_{MA}\varphi'_{MA}) \qquad (4)$$

$$\phi_A = \frac{1}{\sqrt{2}}(\varphi_{MS}\varphi'_{MA} - \varphi_{MA}\varphi'_{MS}). \qquad (5)$$

The factor $1/\sqrt{2}$ is normalization. We can also form mixed symmetric functions ϕ_{MS} and ϕ_{MA}:

$$\phi_{MS} = \frac{1}{\sqrt{2}}(-\varphi_{MS}\varphi'_{MS} + \varphi_{MA}\varphi'_{MA}) \qquad (6)$$

$$\phi_{MA} = \frac{1}{\sqrt{2}}(\varphi_{MS}\varphi'_{MA} + \varphi_{MA}\varphi'_{MS}). \qquad (7)$$

We are now able to write down the structure of the four color octet wave functions, that lie in the 816 dimensional representation of $SU(18)$ [18]. f, s, c denote the flavor, spin and the color three quark wave functions, respectively.

$$
\begin{aligned}
{}^{\mathbf{4}}\mathbf{8} &: \frac{1}{\sqrt{2}}(c_{MS}f_{MA} - c_{MA}f_{MS})s_S && (8)\\
{}^{\mathbf{2}}\mathbf{10} &: \frac{1}{\sqrt{2}}(c_{MS}s_{MA} - c_{MA}s_{MS})f_S && (9)\\
{}^{\mathbf{2}}\mathbf{8} &: \frac{1}{2}\Big[c_{MS}(f_{MA}s_{MS} + f_{MS}s_{MA}) - c_{MA}(f_{MA}s_{MA} - f_{MS}s_{MS})\Big] && (10)\\
{}^{\mathbf{2}}\mathbf{1} &: \frac{1}{\sqrt{2}}(c_{MA}s_{MA} + c_{MS}s_{MS})f_A. && (11)
\end{aligned}
$$

2.2 Construction of hybrid wave functions

We are interested in the flavor singlet sector of the qqq color octet. This is the wave function from eqn. 11:

$$
{}^{\mathbf{2}}\mathbf{1} \;:\; \frac{1}{\sqrt{2}}(c_{MA}s_{MA} + c_{MS}s_{MS})f_A. \tag{12}
$$

This wave function carries a suppressed color index that gets contracted with the color index of the gluon when forming the $qqqg$-state. Only the spin combination is left, as the gluon is flavorless.

The free gluon is a massless particle with two helicity states. In confining the gluon inside the hybrid, it is assumed [5, 6] that the gluon gains mass and with that a third degree of freedom, the spin 0 state. In the limit of zero coupling, the problem can be modeled by the familiar problem in classical electrodynamics to find the cavity normal modes for a massless photon field in a rigid spherical cavity [10]. The confined field is classified by TM or TE modes, with defined total angular momentum J and parity P:

$$
\begin{aligned}
TE &: J^P = 1^+, 2^-, \ldots \\
TM &: J^P = 1^-, 2^+, \ldots\ .
\end{aligned} \tag{13}
$$

(With the assumed boundary conditions, and standard bag model parameters, the TM(1^-) mode is more energetic than the lowest energy eigenmode TE(1^+).) We can construct the quantum numbers of the hybrid state with these J^P values. With the $J = 1$ gluon mode and the flavor singlet three quark state $J = 1/2$, we can form two $qqqg$-states, one with spin 1/2 and one with spin 3/2. In the latter case, gluon spin and qqq spin are aligned, in the former, they are opposite. This will give the main contribution to the mass splittings between these two states.

3 The interaction Hamiltonian

In this section we discuss the Hamiltonian that we use to determine mass splittings of hybrid baryons. The hyperfine interaction Hamiltonian V_{hyp} can be divided into two parts:

$$
V_{hyp} = V_{qq} + V_{qg}. \tag{14}
$$

The first part V_{qq} includes the interaction only between the quarks. The second part V_{qg} contains the interaction between the quarks and the constituent gluon.

The interaction V_{qq} between the three quarks is proportional to [6]

$$V_{qq} \propto -\sum_{i<j} S^i \cdot S^j F^i \cdot F^j. \tag{15}$$

S^i and F^i are the spin and color matrices for the ith quark. Quarks are in the fundamental representation of $SU(2)_S$ and $SU(3)_C$. So the spin and color matrices for a quark are defined by

$$S = \frac{1}{2}\sigma, \quad F = \frac{1}{2}\lambda. \tag{16}$$

σ are the Pauli and λ are the Gell-Mann matrices, given in appendix A. We define the dot products $S \cdot S$ and $F \cdot F$ in appendix B. This effective hyperfine interaction can be interpreted as the dominant contribution from one-gluon exchange between quarks inside the baryon with a radius $r \approx 1\ fm$.

The effective one-gluon ansatz leads to the quark-gluon interaction [6]

$$V_{qg} \propto -\sum_{i} S^i \cdot S^g F^i \cdot F^g, \tag{17}$$

where S^g and F^g are the spin and color matrices for the gluon. In ref. [20] a similar system is studied: baryons coupled to a gluino ($\tilde{g}$), the supersymmetric fermionic partner of the gluon, and many of those techniques can be used here. We however assume different coupling strengths[1] between the light quarks, a light and a heavy quark, a gluon and a light quark and a gluon and a heavy quark which breaks the $SU(3)_F$ symmetry. We label them κ, κ_i, κ_g and κ_{ig} respectively, with i the index for the heavy quark (s, c, b). κ and κ_i can be determined from ordinary baryon mass splittings. This is done in section 3.3.

Our final interaction Hamiltonian is

$$V_{hyp} = -\sum_{i<j} \kappa_{ij} S^i \cdot S^j F^i \cdot F^j - \sum_{i} \kappa_{ig} S^i \cdot S^g F^i \cdot F^g. \tag{18}$$

The hierarchy of the κ's can be guessed by the form of the "Fermi-Breit-interaction" in QCD for single gluon exchange given by [9]:

$$H_I \propto \frac{F^i \cdot F^j S^i \cdot S^j}{m_i\, m_j}. \tag{19}$$

The effective coupling κ would therefore be expected to be inversely proportional to the product of the masses of the interacting particles:

$$\frac{\kappa_i}{\kappa_j} = \frac{m_j}{m_i}. \tag{20}$$

We assume that the hyperfine coefficient is the product of the color magnetic moments. In going from κ to κ_g we are replacing a light quark color magnetic moment with a gluon

[1] The effective coupling strength in [20] also depends on the radius r of the baryonic state. However, bag model calculations [19] show that the radius changes only of the order of a few percent. Because we assume an effective coupling with an accuracy about ten percent, this effect is negligible, and we use radius independent couplings.

magnetic moment. Since this is the same replacement independent of quark flavor, we would expect that

$$\frac{\kappa}{\kappa_g}=\frac{\kappa_s}{\kappa_{sg}}=\frac{\kappa_c}{\kappa_{cg}}. \tag{21}$$

This reduces the number of parameters in the fit of hybrid baryons and makes it more predictive.

3.1 Quark-quark interactions

In this section we calculate the matrix elements of the interaction Hamiltonian V_{qq} (see eqn. 18). The quark-quark interaction is given by

$$V_{qq} \;=\; -\sum_{i<j}\kappa_{ij}S^i\cdot S^jF^i\cdot F^j. \tag{22}$$

κ_{ij} is the effective coupling between the constituent quarks. The matrix element $E_{hyp} = < V_{qq} >$ has to be evaluated in the quark wave function, which has special symmetry properties.

In section 2.1 we discussed the quark wave function for exact flavor symmetry. We will now include flavor breaking but will assume that isospin is a good symmetry. If we consider $SU(3)_F$ breaking, the wave function of a baryon with two light quarks (u or d) has the structure $\Psi=\Psi^A+\epsilon\Psi^{12}$. The part Ψ^A is antisymmetric under interchange of any two quarks. The part Ψ^{12} is antisymmetric only under interchange of quark 1 and 2 in the basis (see appendix C) in which the light quarks are chosen to be 1 and 2. When $SU(3)_F$ is unbroken, ϵ goes to zero. The measure for flavor breaking is the mass difference between the heavy and the light quark, so ϵ is proportional to m_h-m_l. To evaluate matrix elements of an operator O in state Ψ, we decompose O into a sum of operators which are either totally antisymmetric under interchange of any pair of quarks (O^A) or totally antisymmetric under interchange of the first and second quark (O^{12}):

$$< O > \;=\; < O^A > + < O^{12} > . \tag{23}$$

According to these arguments, we decompose the quark-quark interaction as follows:

$$V_{qq} \;=\; -\kappa_3\underbrace{\sum_{i<j}S^i\cdot S^jF^i\cdot F^j}_{term\,1}-(\kappa-\kappa_3)\underbrace{S^1\cdot S^2F^1\cdot F^2}_{term\,2}. \tag{24}$$

The first term is completely symmetric. Its evaluation has been done in [20]. The authors found with the help of permutation operators for exact flavor symmetry:

$$< O^A > \;=\; <\sum_{i<j}S^i\cdot S^jF^i\cdot F^j>=\frac{21}{16}-\frac{1}{8}C_C^{qqq}-\frac{1}{4}C_F^{qqq}-\frac{1}{12}C_S^{qqq}. \tag{25}$$

The expectation value is written in terms of Casimir operators C (see Appendix B for definition and values in various representations). There is another nice evaluation of (25) by Jaffe [21], involving the $SU(6)$ Casimir operator C_6 of color and spin:

$$< O^A > \;=\; <\sum_{i<j}S^i\cdot S^jF^i\cdot F^j>=\frac{1}{32}C_6^{qqq}-\frac{1}{12}C_S^{qqq}-\frac{1}{8}C_C^{qqq}-\frac{3}{2}. \tag{26}$$

$SU(3)_C$	**8**	**8**	**8**	**8**
$SU(3)_F$	**8**	**8**	**10**	**1**
$SU(2)_S$	**2**	**4**	**2**	**2**
$< O^A >$	1/8	−1/8	−5/8	7/8

$SU(3)_C$	$\bar{\mathbf{3}}$	$\bar{\mathbf{3}}$	**6**	**6**
$SU(2)_I$	**1**	**3**	**1**	**3**
$SU(2)_S$	**1**	**3**	**3**	**1**
$< O^{12} >$	1/2	−1/6	1/12	−1/4

Table 2: Expectation values of $O^A = \sum_{i<j} S^i \cdot S^j F^i \cdot F^j$ and $O^{12} = S^1 \cdot S^2 F^1 \cdot F^2$

The definition and values of C_6 in various representations may be found in the appendix of [22]. We give the values for $< \sum_{i<j} S^i \cdot S^j F^i \cdot F^j >$ for the color octet three quark states in the first part of table 2.

The second term of eqn. 24 is O^{12}. This term is only symmetric under interchange of quark 1 and 2. It has to be evaluated in the wave function with broken flavor symmetry. Using isospin rather than flavor, we can modify eqn. 25 for this case and find:

$$< O^{12} >=< S^1 \cdot S^2 F^1 \cdot F^2 >= \frac{2}{3} - \frac{1}{8} C_C^{12} - \frac{1}{4} C_I^{12} - \frac{1}{12} C_S^{12}. \tag{27}$$

$C_{C,I,S}^{12}$ are the 1-2 diquark Casimir operators for color, isospin and spin, respectively. We give the values for $< S^1 \cdot S^2 F^1 \cdot F^2 >$ for all possible antisymmetric representations in flavor, color and isospin of the diquark in the second part of table 2.

3.2 Quark-gluon interactions

In this section the quark-gluon interaction is evaluated. The quark-gluon interaction is given by (18)

$$V_{qg} \quad = \quad - \sum_i \kappa_{ig} S^i \cdot S^g F^i \cdot F^g. \tag{28}$$

κ_{ig} is the effective coupling between the constituent gluon and the quarks. According to the arguments made in section 3.1, the quark-gluon interaction has to be decomposed as follows:

$$\begin{aligned} V_{qg} \quad = \quad & -\kappa_{3g} \underbrace{\sum_i S^i \cdot S^g F^i \cdot F^g}_{term\,1} \\ & -(\kappa_g - \kappa_{3g}) \underbrace{\left(S^1 \cdot S^g F^1 \cdot F^g + S^2 \cdot S^g F^2 \cdot F^g\right)}_{term\,2}. \end{aligned} \tag{29}$$

The first term is completely symmetric under interchange of any pair of quarks. The second term is symmetric only under interchange of quark 1 and 2.

The computation of the matrix elements of the first term has already been done for the light flavor octets and decuplets by Barnes and Close [5]. We list our new results for the flavor singlet and for completeness, the Barnes and Close results for the light octets and decuplets, in table 3. For the benefit of the reader we give a sample computation for the flavor singlet in Appendix D.

For the light color octets and light decuplets, term 2 does not contribute at all because $\kappa_g = \kappa_{3g}$. The evaluation of this term is more delicate for hybrids which contain one heavy

${}^{spin}flavor$	$^{4}\mathbf{8}$			$^{2}\mathbf{10}$		$^{2}\mathbf{8}$		$^{2}\mathbf{1}$	
total J	5/2	3/2	1/2	3/2	1/2	3/2	1/2	3/2	1/2
$< \sum_i S^i \cdot S^g F^i \cdot F^g >$	-3/2	1	5/2	0	0	-1/2	1	-1	2
$< S^1 \cdot S^g F^1 \cdot F^g >$	I=1			I=1		I=1		I=0	
	-3/8	1/4	5/8	-1/4	1/2	-1/4	1/2	0	0
	I=0					I=0			
	-5/8	5/12	25/24			0	0		

Table 3: Values for $< \sum_i S^i \cdot S^g F^i \cdot F^g >$ and $< S^1 \cdot S^g F^1 \cdot F^g >$

quark i. As we assume that flavor is broken, but that isospin is still a good symmetry, isospin singlet ($I = 0$) and isospin triplet ($I = 1$) hybrid baryons arise[2]. For the hybrid states we find

$$^{2}\mathbf{1}(I=0), \qquad ^{2}\mathbf{10}(I=1), \qquad ^{4}\mathbf{8}(I=0,1), \qquad ^{2}\mathbf{8}(I=0,1). \tag{30}$$

For ${}^{spin}flavor$ fixed, hybrids within isospin multiplets have the same mass, and hybrids between isospin multiplets have different mass. In order to compute term 2 we use the symmetry under interchange of quark 1 and 2:

$$< S^1 \cdot S^g F^1 \cdot F^g > \; = \; < S^2 \cdot S^g F^2 \cdot F^g > . \tag{31}$$

We can write

$$\begin{aligned} &< S^1 \cdot S^g F^1 \cdot F^g > \\ &= \frac{1}{4}\left[(S^1+S^g)^2 - (S^1)^2 - (S^g)^2\right]\left[(F^1+F^g)^2 - (F^1)^2 - (F^g)^2\right] \\ &= \frac{1}{4}\left[(S^1+S^g)^2 - \frac{11}{4}\right]\left[(F^1+F^g)^2 - \frac{13}{3}\right]. \end{aligned} \tag{32}$$

What remains is to know the spin and color representations of the gluon–first quark state ($S^1 + S^g$ and $F^1 + F^g$) of each hybrid baryon. In order to expand the color and spin wave function of each hybrid (with the help of $SU(2)$ [1] and $SU(3)$ [23] Clebsch Gordan coefficients) into gluon–first quark and second quark–third quark color and spin wave functions, we need to know the spin and color representations of the diquark. The 1-2 symmetric part of each hybrid wave function has to fulfill two constraints. Firstly, the flavor part must be f_{MS} (or f_S) for isotriplets and f_{MA} (or f_A) for isosinglets. Secondly, the wave function must be antisymmetric under interchange of quark 1 and 2. The wave functions for the $^{4}\mathbf{8}$ hybrids result immediately from (8):

$$^{4}\mathbf{8}(I=1) \;:\; f_{MS} c_{MA} s_S, \tag{33}$$

$$^{4}\mathbf{8}(I=0) \;:\; f_{MA} c_{MS} s_S. \tag{34}$$

[2]For the quark content uds, the hypercharge is zero. Thus, the I_3 equals the electric charge of the hybrids. We could imagine to measure the isospin experimentally by measuring the charge of the hybrids. For isotriplets we expect three mass degenerate hybrids with charge -,0,+ and for the isosinglet a single hybrid with charge 0 but different mass.

For the other hybrids (see (9) - (11)), the wave function of each hybrid may be a linear combination (lc) of the following functions:

$$\begin{aligned} {}^{\mathbf{2}}\mathbf{1}(I=0) &\;:\; lc\ of\ f_A c_{MS} s_{MS}\ and\ f_A c_{MA} s_{MA}, && (35)\\ {}^{\mathbf{2}}\mathbf{10}(I=1) &\;:\; lc\ of\ f_S c_{MS} s_{MA}\ and\ f_S c_{MA} s_{MS}, && (36)\\ {}^{\mathbf{2}}\mathbf{8}(I=1) &\;:\; lc\ of\ f_{MS} c_{MS} s_{MA}\ and\ f_{MS} c_{MA} s_{MS}, && (37)\\ {}^{\mathbf{2}}\mathbf{8}(I=0) &\;:\; lc\ of\ f_{MA} c_{MS} s_{MS}\ and\ f_{MA} c_{MA} s_{MA}. && (38)\end{aligned}$$

All the above parts of the wave functions are eigenfunctions of the operator

$$A \;=\; -(\kappa-\kappa_3)\,S^1\cdot S^2 F^1\cdot F^2 - 2(\kappa_g-\kappa_{3g})\,S^1\cdot S^g F^1\cdot F^g, \tag{39}$$

which is the complete 1-2 symmetric part of our interaction Hamiltonian V_{hyp} (14). We assume that the diquark is in a state in which the energy is minimal, i.e., in which A is minimized. The wave functions with minimal energy are:

$$\begin{aligned} {}^{\mathbf{2}}\mathbf{1}(I=0) &\;:\; f_A c_{MA} s_{MA}, && (40)\\ {}^{\mathbf{2}}\mathbf{10}(I=1) &\;:\; f_S c_{MA} s_{MS}, && (41)\\ {}^{\mathbf{2}}\mathbf{8}(I=1) &\;:\; f_{MS} c_{MA} s_{MS}, && (42)\\ {}^{\mathbf{2}}\mathbf{8}(I=0) &\;:\; f_{MA} c_{MA} s_{MA}. && (43)\end{aligned}$$

The eigenvalues of A were computed with the estimate values (in MeV) $\kappa = 290$, $\kappa_3 = 180$, $\kappa_g = 60$ and $\kappa_{3g} = 40$. We list the resulting values for $< S^1 \cdot S^g F^1 \cdot F^g >$ in table 3. Values for $< S^1 \cdot S^2 F^1 \cdot F^2 >$ can be taken from the second part of table 2.

3.3 The hyperfine coupling constant κ

We fit the mass splittings of ordinary baryons to find values for κ, κ_s and κ_c. For the fit we use the masses of the isospin multiplets given in table 4. We use the average mass of each isospin multiplet. (The members within isospin multiplets stay degenerate since we assume exact isospin symmetry and neglect electroweak interactions, which are of the order of 0.5%). The listed isospin multiplets Σ_i^*, Σ_i and Λ_i are labeled by the flavor index of the heavy quark $i = s, c, b$. If we assumed exact $SU(3)_F \times SU(2)_S$ symmetry, the Σ_i^*, Σ_i and Λ_i (for i fixed) would be members of the totally symmetric 56-dimensional representation. (compare eqn. (2)). By switching on the mass-difference between the heavy quark i and the light quarks q we break $SU(3)_F$. The values of κ, κ_s and κ_c can thus be determined by the experimentally observed mass separations of the isospin multiplets. Calculations for mass splittings of this kind are summarized in the book by Close [18], p. 387. We obtain the same values of the matrix elements (defined in eqns. 25 and 27) for the quark-quark interaction (eqn. 24)

$$V_{qq} \;=\; -\kappa_3 \sum_{i<j} S^i \cdot S^j F^i \cdot F^j - (\kappa-\kappa_3)\,S^1\cdot S^2 F^1 \cdot F^2, \tag{44}$$

and list the values for $E_{hyp} =< V_{qq} >$ in table 5.

With these values at hand, we find for the mass splittings:

κ_i	particle	spin	isospin	flavor	content	$\Delta M/MeV$	fit	%	κ_i/MeV
κ	$\Delta(1232)$	3/2	3/2	**10**	qqq	$\Delta - N = 293$	293		293
	$N(939)$	1/2	1/2	**8**	qqq				
κ_s	$\Sigma_s^*(1385)$	3/2	1	**10**	qqs	$\Sigma_s^* - \Sigma_s = 192$	182	5	182
	$\Sigma_s(1193)$	1/2	1	**8**	qqs	$\Sigma_s - \Lambda_s = 77$	74	4	
	$\Lambda_s(1116)$	1/2	0	**8**	qqs	$\Sigma_s^* - \Lambda_s = 269$	256	5	
κ_c	$\Sigma_c^*(2520)$	3/2	1	**10**	qqc	$\Sigma_c^* - \Sigma_c = 65$	60	8	60
	$\Sigma_c(2455)$	1/2	1	**8**	qqc	$\Sigma_c - \Lambda_c = 170$	155	9	
	$\Lambda_c(2285)$	1/2	0	**8**	qqc	$\Sigma_c^* - \Lambda_c = 235$	215	8	
κ_b	$\Sigma_b^*(?)$	3/2	1	**10**	qqb	$\Sigma_b^* - \Sigma_b = ?$	18	20	18
	$\Sigma_b(?)$	1/2	1	**8**	qqb	$\Sigma_b - \Lambda_b = ?$	183	20	
	$\Lambda_b(5640)$	1/2	0	**8**	qqb	$\Sigma_b^* - \Lambda_b = ?$	201	20	

Table 4: The particles used to determine κ_i

particle	flavor	diquark		$< \sum_{i<j} S^i \cdot S^j F^i \cdot F^j >$	$< S^1 \cdot S^2 F^1 \cdot F^2 >$	E_{hyp}
Σ_i^*	**10**	$C_S^{12}\{\mathbf{3}\}$	$C_I^{12}\{\mathbf{3}\}$	-1/2	-1/6	$\frac{1}{6}\kappa + \frac{1}{3}\kappa_3$
Σ_i	**8**	$C_S^{12}\{\mathbf{3}\}$	$C_I^{12}\{\mathbf{3}\}$	1/2	-1/6	$\frac{1}{6}\kappa - \frac{2}{3}\kappa_3$
Λ_i	**8**	$C_S^{12}\{\mathbf{1}\}$	$C_I^{12}\{\mathbf{1}\}$	1/2	1/2	$-\frac{1}{2}\kappa$

Table 5: Values for E_{hyp}

$$
\begin{aligned}
E(\Sigma_i^*) - E(\Sigma_i) &= \kappa_i, && (45)\\
E(\Sigma_i) - E(\Lambda_i) &= \frac{2}{3}\kappa - \frac{2}{3}\kappa_i, && (46)\\
E(\Sigma_i^*) - E(\Lambda_i) &= \frac{2}{3}\kappa + \frac{1}{3}\kappa_i. && (47)
\end{aligned}
$$

With increasing mass of the heavy quark i, κ_i goes down (20) so that the $\Sigma_i^* - \Sigma_i$ splittings go down, the $\Sigma_i - \Lambda_i$ splittings go up and the $\Sigma_i^* - \Lambda_i$ splittings go down. This effect can be observed in table 4, where we included the results for our κ_i fits on the right side.

The masses of the Σ_b^* and Σ_b have not yet been measured. We can predict the order of their mass splittings. We use relation (20)

$$\kappa_b = \frac{m_c}{m_b}\kappa_c \qquad (48)$$

to find an estimate value for κ_b. With values $m_c = (1.25 \pm 0.15)$ GeV and $m_b = (4.25 \pm 0.15)$ GeV from [1], we find e.g. for:

$$\Sigma_b^*(?) - \Sigma_b(?) \quad = \quad (18 \pm 4)\, MeV = \kappa_b. \qquad (49)$$

The errors result from the uncertainty of the quark masses. Errors arising due to assumption (20) are not included.

4 Masses and mass splittings of the hybrid baryons

In this section we determine the masses and mass splittings of the flavor octet and decuplet hybrid baryons which contain two light quarks and one heavy quark i. If the hyperfine interaction V_{hyp} (18) would be absent, the ${}^{spin}flavor$ states $\mathbf{{}^21}$, $\mathbf{{}^28}$, $\mathbf{{}^48}$ and $\mathbf{{}^210}$ which form the **816** in eq. (1), would be degenerate and would have the common mass E_{0i}. If V_{hyp} is present, the mass of each hybrid is given by

$$E_i = E_{0i} + E_{hyp}. \tag{50}$$

We neglect a possible mixing of the hybrids with other states which carry the same quantum numbers.

In section 4.1 we determine κ_g and κ_{ig} from the splittings of the flavor singlet hybrids and calculate E_{0i}. In sections 4.2 and 4.3 we give the absolute masses E_i (50) for the flavor octet and decuplet hybrid baryons.

4.1 Mass splittings of the flavor singlet hybrid baryons

The effective couplings κ_{ig} can be determined from the mass splittings of the flavor singlet hybrid baryons. We briefly discuss the expected spectrum of the hybrids (compare section 2). The three quarks have to form a flavor singlet and a color octet. So their spin state has to be a doublet. When the $\mathbf{{}^21}$ three quark state couples to the constituent gluon (spin triplet), a $J=\frac{1}{2}$ and a $J=\frac{3}{2}$ hybrid state are formed. From eqn. 24, eqn. 29 and tables 2 and 3 we find:

$$E_{hyp} \;\; = \;\; -\frac{7}{8}\kappa_3 - \frac{1}{2}(\kappa-\kappa_3) - \kappa_{3g}\left\{ \begin{matrix} -1 \\ 2 \end{matrix} \right\} \begin{matrix} J=\frac{3}{2} \\ J=\frac{1}{2} \end{matrix} \tag{51}$$

The mass separation is

$$E_{hyp}(J=3/2) - E_{hyp}(J=1/2) \;\; = \;\; 3\,\kappa_{3g}. \tag{52}$$

For the strange and the charm system we have

$$\Lambda_s(1520) - \Lambda_s(1405) \;\; = \;\; 115\,MeV = 3\,\kappa_{sg} \tag{53}$$

$$\Lambda_c(2625) - \Lambda_c(2593) \;\; = \;\; 32\,MeV = 3\,\kappa_{cg}. \tag{54}$$

So the values for κ_{sg} and κ_{cg} are determined:

$$\kappa_{sg} \;\; = \;\; 38\,MeV \tag{55}$$

$$\kappa_{cg} \;\; = \;\; 11\,MeV. \tag{56}$$

We can estimate the mass separation between the $\Lambda_b(J=3/2)$ and the $\Lambda_b(J=1/2)$. We use (20) and (21)

$$\kappa_{bg} = \frac{m_c}{m_b}\kappa_{cg} \tag{57}$$

to find an estimate value for κ_{bg}. We use the masses $m_c = (1.25 \pm 0.15)$ GeV and $m_b = (4.25 \pm 0.15)$ GeV [1] and find $\kappa_{bg} = (3 \pm 0.5)$ MeV. It follows

$$\Lambda_b(J=3/2) - \Lambda_b(J=1/2) \;\; = \;\; (9 \pm 2)\,MeV. \tag{58}$$

Using these values for κ_{sg}, κ_{cg} and the relation from eqn. 21 we find for $i = s$ and $i = c$ respectively

$$\begin{array}{lllllll} \kappa_g & = & \kappa_{sg}\frac{\kappa}{\kappa_s} & = & 38\frac{293}{182}\,MeV & = & 61\,MeV, \\ \kappa_g & = & \kappa_{cg}\frac{\kappa}{\kappa_c} & = & 11\frac{293}{60}\,MeV & = & 54\,MeV. \end{array} \tag{59}$$

This corresponds to a value of κ_g (with a relative error less than 10 %) of

$$\kappa_g \quad = \quad (58 \pm 4)\,MeV. \tag{60}$$

The error of κ_g is sufficient for the predictive power of our effective model. Having gained all values for the various κ_i, we can determine E_{hyp} for $\Lambda_s(E_s = 1405)$ and $\Lambda_c(E_c = 2593)$ given in (51):

$$E_{hyp}(\Lambda_s) = -291\ MeV; \qquad E_{hyp}(\Lambda_c) = -191\ MeV. \tag{61}$$

From (50) we find

$$E_{0s} = 1696\ MeV; \qquad E_{0c} = 2784\ MeV. \tag{62}$$

4.2 Mass splittings of the flavor octet hybrid baryons

The three quarks have to form a flavor and a color octet. So their spin state may be a doublet or a quartet. There are the $^{spin}flavor$ states $\mathbf{^28}$ and $\mathbf{^48}$, respectively. When the $\mathbf{^28}$ three quark state couples to the constituent gluon (spin triplet), a $J = \frac{1}{2}$ and a $J = \frac{3}{2}$ hybrid state is formed. When the $\mathbf{^48}$ three quark state couples to the constituent gluon, a $J = \frac{1}{2}$, $J = \frac{3}{2}$ and a $J = \frac{5}{2}$ hybrid state is formed.

4.2.1 Splittings in the $^{\mathbf{spin}}$flavor $=^{\mathbf{2}}$ 8 sector

From eqn. 24, eqn. 29 and tables 2 and 3 we find:

$$E_{hyp}(I=1) \quad = \quad -\frac{1}{8}\kappa_3 + \frac{1}{6}(\kappa - \kappa_3) - \kappa_{3g}\left\{\begin{array}{c} -1/2 \\ 1 \end{array}\right\} - 2(\kappa_g - \kappa_{3g})\left\{\begin{array}{c} -1/4 \\ 1/2 \end{array}\right\} \begin{array}{l} J=\frac{3}{2} \\ J=\frac{1}{2} \end{array} \tag{63}$$

$$E_{hyp}(I=0) \quad = \quad -\frac{1}{8}\kappa_3 - \frac{1}{2}(\kappa - \kappa_3) - \kappa_{3g}\left\{\begin{array}{c} -1/2 \\ 1 \end{array}\right\} - 2(\kappa_g - \kappa_{3g})\left\{\begin{array}{c} 0 \\ 0 \end{array}\right\} \begin{array}{l} J=\frac{3}{2} \\ J=\frac{1}{2} \end{array} \tag{64}$$

We list the resulting absolute masses E_i (50) in table 6.

4.2.2 Splittings in the $^{\mathbf{spin}}$flavor $=^{\mathbf{4}}$ 8 sector

From eqn. 24, eqn. 29 and tables 2 and 3 we find:

$$E_{hyp}(I=1) = +\frac{1}{8}\kappa_3 + \frac{1}{6}(\kappa - \kappa_3) - \kappa_{3g}\left\{\begin{array}{c} -3/2 \\ 1 \\ 5/2 \end{array}\right\} - 2(\kappa_g - \kappa_{3g})\left\{\begin{array}{c} -3/8 \\ 1/4 \\ 5/8 \end{array}\right\} \begin{array}{l} J=5/2 \\ J=3/2 \\ J=1/2 \end{array} \tag{65}$$

$$E_{hyp}(I=0) = +\frac{1}{8}\kappa_3 - \frac{1}{12}(\kappa - \kappa_3) - \kappa_{3g}\left\{\begin{array}{c} -3/2 \\ 1 \\ 5/2 \end{array}\right\} - 2(\kappa_g - \kappa_{3g})\left\{\begin{array}{c} -5/8 \\ 5/12 \\ 25/24 \end{array}\right\} \begin{array}{l} J=5/2 \\ J=3/2 \\ J=1/2 \end{array} \tag{66}$$

(67)

We list the resulting absolute masses E_i (50) in table 6.

4.3 Mass splittings of the flavor decuplet hybrid baryons

The three quarks form a flavor decuplet and a color octet. So their spin state must be a doublet to ensure antisymmetry under permutations. When the $^{2}\mathbf{10}$ three quark state couples to the constituent gluon (spin triplet), a $J = \frac{1}{2}$ and a $J = \frac{3}{2}$ state are formed. From eqn. 24, eqn. 29 and tables 2 and 3 we find:

$$E_{hyp}(I=1) \quad = \quad +\frac{5}{8}\kappa_3 + \frac{1}{6}(\kappa - \kappa_3) - \kappa_{3g}\begin{Bmatrix} 0 \\ 0 \end{Bmatrix} - 2(\kappa_g - \kappa_{3g})\begin{Bmatrix} -1/4 \\ 1/2 \end{Bmatrix}\begin{matrix} J=\frac{3}{2} \\ J=\frac{1}{2} \end{matrix} \qquad (68)$$

We list the resulting absolute masses E_i (50) in table 6. (We also give the mass splittings ΔE_b for all the beauty hybrids).

$^{spin}flavor$	J^P	E_s/MeV	E_c/MeV	$\Delta E_b/MeV$
$^{4}\mathbf{8}$	$5/2^-$	1809	2882	$E(5/2) - E(3/2) = 76$
(I=1)	$3/2^-$	1689	2796	$E(3/2) - E(1/2) = 46$
	$1/2^-$	1617	2744	
$^{4}\mathbf{8}$	$5/2^-$	1792	2847	$E(5/2) - E(3/2) = 123$
(I=0)	$3/2^-$	1655	2722	$E(3/2) - E(1/2) = 73$
	$1/2^-$	1573	2647	
$^{2}\mathbf{8}$	$3/2^-$	1721	2844	$E(3/2) - E(1/2) = 87$
(I=1)	$1/2^-$	1634	2757	
$^{2}\mathbf{8}$	$3/2^-$	1637	2666	$E(3/2) - E(1/2) = 5$
(I=0)	$1/2^-$	1580	2649	
$^{2}\mathbf{10}$	$3/2^-$	1838	2884	$E(3/2) - E(1/2) = 83$
(I=1)	$1/2^-$	1808	2813	

Table 6: Masses of the flavor decuplet and flavor octet hybrids

5 The parity of $\Lambda(1405)$

We assumed so far, that the parity of $\Lambda(1405)$ is negative as listed in PDG [1]. This choice is made because in the quark model, the $\Lambda(1405)$ necessarily needs to be orbitally excited and thus must have negative parity. However, there is no direct evidence from experiment that $\Lambda(1405)$ actually has negative parity. Hemingway [26] writes, that a "Byers-Fenster spin-parity analysis gives no parity discrimination". Thomas [27] writes, that "the experimental facts are that the parity of $\Lambda(1405)$ has not yet been determined in a production experiment" and that they were "unable to make a parity determination". We therefore regard the parity of the $\Lambda(1405)$ as experimentally undetermined. If the $\Lambda(1405)$ is the lightest hybrid baryon, the hybrid model strongly suggests it actually has even parity because the bag model predicts [18] the lightest $J^P = 1^{(-)}(TM)$ gluon mode is about 300 MeV heavier than the lightest $J^P = 1^{(+)}(TE)$ gluon mode. The negative parity partner of the $\Lambda(1405)$ hybrid would thus be about 300 MeV heavier with mass of about 1.7 GeV. It's experimental detection would be more difficult due to mixing with other states. A second possibility is that the $\Lambda(1405)$ has odd parity and there is a lower

mass pair of even parity states. If the shift is about 300 MeV as predicted from bag models, there would be $J^P = \frac{1}{2}^{(+)}, \frac{3}{2}^{(+)}$ states with strangeness -1 and mass about 1.1 and 1.2 GeV. It is doubtful to us that such low energy states could be discovered by partial wave analysis. They would be far below threshold in the $\Sigma\ \pi$ channel. A positive parity $\Lambda(1100)$ would only impact the L=1 state and furthermore would have little impact compared to the $\Lambda(1405)$ which is much closer to the physical region.

6 A low lying dihyperon ?

We are exploring the ansatz that a *uds* in a color octet, flavor singlet state binds with a constituent gluon to produce the $\Lambda(1405)$. We analyzed mass splittings between members of the various multiplets, but we made no absolute mass predictions. (Those are model dependent and would require the use of a model such as the Skyrme, MIT bag or potential models). However, if we make the hybrid baryon ansatz for these states, we know the masses experimentally. Lattice calculations [28] give the lightest glueball in the range 1.4 - 1.7 GeV, where there are good glueball candidates. Thus if the hybrid baryon ansatz is correct, the approximate coincidence of the *udsg* and *gg* masses suggests that a *uds* in a flavor singlet color octet state is approximately equivalent to a gluon from the dynamical point of view. The dynamics of a hadronic bound state depends primarily on the color, mass and spin of the constituents. Thus a spatially-compact *uds* system in a flavor singlet state would behave like a gluon with spin 1/2 rather than spin 1. Thus the dynamical similarity of the *uds* and gluon can only be approximate and mass estimates must have about a 100 MeV uncertainty at least. Making this ansatz, a combination of two *uds* should be a glueball-like state with mass also of about 1.5 GeV. This would be the H dihyperon, which is a six quark state with total spin and isospin zero, baryon number 2 and strangeness -2. The dihyperon was predicted in 1977 by Jaffe [21] in a MIT bag model calculation with mass of 2150 MeV. Since that, many other dihyperon mass calculations have been performed, also using Skyrme [29], [30] and quark cluster models [31]. The mass estimates for the lowest lying dibaryon H range from 1.5 to 2.2 GeV. The differences in the mass predictions are attributed to the difference between the models which are characterized by model parameters and the model dependent assumptions which are made in order to describe hadronically bound states. If the $\Lambda(1405)$ is a hybrid baryon, we therefore suspect that the mass of the H dibaryon is in the 1.5 GeV mass region, as references [29] suggest. We leave to another work a discussion of the phenomenological issues and detectability of such a light H.

7 Summary and conclusion

We have explored the hypothesis that four particles $\Lambda_s(1405)$, $\Lambda_s(1520)$, $\Lambda_c(2593)$ and $\Lambda_c(2676)$, are hybrids. The observed mass splittings are consistent with the hybrid baryon hypothesis, resolving a severe problem of the conventional identification as an orbital excitation of a 3 quark state. It is non-trivial that the ordering of states is $m_{J=3/2} > m_{J=1/2}$, as observed experimentally; in the conventional $L = 1$ picture the spin-3/2 state is necessarily the lightest. Assuming these states are flavor singlet hybrids fixes the parameters of the quark-gluon hyperfine interaction. This allows the mass splittings of the flavor octet and decuplet hybrid baryons to be predicted, but without developing a

theory of mixing with nearby ordinary octets and decuplets these predictions cannot be tested.

The best test of the ansatz that the $\Lambda_s(1405)$, $\Lambda_s(1520)$, $\Lambda_c(2593)$ and $\Lambda_c(2676)$, are hybrids is that they will be parity doubled, with the odd parity partner about 300 MeV heavier than the even parity state. Thus we predict either that the $\Lambda_s(1405)$, $\Lambda_s(1520)$, $\Lambda_c(2593)$ and $\Lambda_c(2676)$ are even parity, or that there are as-yet-undiscovered even parity flavor singlet, strangeness -1 states at about 1.1 and 1.2 GeV. The hybrid ansatz suggests, but does not predict, that the H dibaryon mass is around 1.5 GeV.

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A The Pauli and the Gell-Mann matrices

The Pauli matrices [18] p. 23 are chosen to be

$$\sigma_1 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \quad \sigma_2 = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}, \quad \sigma_3 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

The Gell-Mann matrices [18] p. 30 are given by

$$\lambda_1 = \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \quad \lambda_2 = \begin{pmatrix} 0 & -i & 0 \\ i & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \quad \lambda_3 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

$$\lambda_4 = \begin{pmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \end{pmatrix} \quad \lambda_5 = \begin{pmatrix} 0 & 0 & -i \\ 0 & 0 & 0 \\ i & 0 & 0 \end{pmatrix} \quad \lambda_6 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}$$

$$\lambda_7 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & -i \\ 0 & i & 0 \end{pmatrix} \quad \lambda_8 = \frac{1}{\sqrt{3}} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -2 \end{pmatrix}$$

B Casimir operators

The Casimir operators $C_{2,3}$ [24] pp. 88, 89 are defined:

$$SU(2): \quad C_2 = S \cdot S = \sum_{m=1}^{3} S_m S_m = \frac{1}{4} \sum_{m=1}^{3} \sigma_m \sigma_m \tag{69}$$

$$S_m = \frac{1}{2} \sigma_m \tag{70}$$

$$SU(3): \quad C_3 = F \cdot F = \sum_{a=1}^{8} F_a F_a = \frac{1}{4} \sum_{a=1}^{8} \lambda_a \lambda_a \tag{71}$$

$$F_a = \frac{1}{2} \lambda_a \tag{72}$$

The values of these operators in various representations are given in table 7. We use $C_2 = S \cdot (S+1)$. The values for C_3 can be found in [25] p. 74.

$SU(2)$:

dim	**1**	**2**	**3**	**4**	**5**
C_2	0	3/4	2	15/4	6

$SU(3)$:

dim	**1**	**3**	**6**	**8**	**10**
C_3	0	4/3	10/3	3	6

Table 7: Values for $SU(2)$ and $SU(3)$ Casimir operators in various representations

C Definitions of the mixed symmetry functions

The φ_{MS} and φ_{MA} are defined in the table 8 . This table is taken from [18] p. 46. These states are mixed symmetric or mixed antisymmetric. They are symmetric or antisymmetric under interchange of the first two quarks. The various wave functions for $SU(3)$ are named by the name of the particle they belong to.

D Sample calculation for the flavor singlet

In this appendix we give an example of calculating the first term of V_{qg} in eqn. 29. To evaluate the first term, we need to look at the structure of the completely antisymmetric three quark wave function, discussed in section 2. It follows for the flavor singlet state $^{\mathbf{2}}\mathbf{1}$:

$$\begin{aligned} &< \sum_i S^i \cdot S^g F^i \cdot F^g >= 3 < S^3 \cdot S^g F^3 \cdot F^g >= \\ &= \frac{3}{2} \Big\{ < S^3 \cdot S^g >_{MA} < F^3 \cdot F^g >_{MA} + < S^3 \cdot S^g >_{MS} < F^3 \cdot F^g >_{MS} \Big\}. \end{aligned} \tag{73}$$

(The first line follows from the antisymmetry.) The subscripts MA and MS, introduced in section 2.1, indicate that the pair of the first and second quark is an antisymmetric or

label	φ_{MS}	φ_{MA}
P	$\frac{1}{\sqrt{6}}\left[(ud+du)u-2uud\right]$	$\frac{1}{\sqrt{2}}(ud-du)u$
N	$-\frac{1}{\sqrt{6}}\left[(ud+du)d-2ddu\right]$	$\frac{1}{\sqrt{2}}(ud-du)d$
Σ^+	$\frac{1}{\sqrt{6}}\left[(us+su)u-2uus\right]$	$\frac{1}{\sqrt{2}}(us-su)u$
Σ^0	$\frac{1}{\sqrt{6}}\left[s\left(\frac{du+ud}{\sqrt{2}}\right)+\left(\frac{dsu+usd}{\sqrt{2}}\right) -2\left(\frac{du+ud}{\sqrt{2}}\right)s\right]$	$\frac{1}{\sqrt{2}}\left[\left(\frac{dsu+usd}{\sqrt{2}}\right)-s\left(\frac{ud+du}{\sqrt{2}}\right)\right]$
Σ^-	$\frac{1}{\sqrt{6}}\left[(ds+sd)d-2dds\right]$	$\frac{1}{\sqrt{2}}(ds-sd)d$
Λ^0	$\frac{1}{\sqrt{2}}\left[\left(\frac{dsu-usd}{\sqrt{2}}\right)+\frac{s(du-ud)}{\sqrt{2}}\right]$	$\frac{1}{\sqrt{6}}\left[\frac{s(du-ud)}{\sqrt{2}}+\frac{usd-dsu}{\sqrt{2}} -\frac{2(du-ud)s}{\sqrt{2}}\right]$
Ξ^-	$-\frac{1}{\sqrt{6}}\left[(ds+sd)s-2ssd\right]$	$\frac{1}{\sqrt{2}}(ds-sd)s$
Ξ^0	$-\frac{1}{\sqrt{6}}\left[(us+su)s-2ssu\right]$	$\frac{1}{\sqrt{2}}(us-su)s$

Table 8: The functions of mixed symmetry

a symmetric state, respectively. In the color case this means for $< F^3 \cdot F^g >_{MA}$ that the first and second quark are in a color antitriplet, $\bar{\mathbf{3}}$. So the gluon-third quark system has to be in a color triplet to form an overall singlet with the spectator diquark. Similarly for $< F^3 \cdot F^g >_{MS}$, the first and second quark form a symmetric state, the color sextet. Thus the gluon and the third quark are an antisextet. For the flavor singlet with total $J = 1/2$ we have for the color operator:

$$
\begin{aligned}
< F^3 \cdot F^g >^{MS}_{MA} &= \frac{1}{2}\Big[(F^3+F^g)^2-(F^3)^2-(F^g)^2\Big] && (74)\\
&= \frac{1}{2}\Big[\begin{Bmatrix} C_F^{3g}\{\bar{\mathbf{6}}\} \\ C_F^{3g}\{\mathbf{3}\}\end{Bmatrix} - C_F^q\{\mathbf{3}\} - C_F^g\{\mathbf{8}\}\Big] && (75)\\
&= \frac{1}{2}\left[\begin{Bmatrix} 10/3 \\ 4/3\end{Bmatrix} - \frac{4}{3} - 3\right] = \begin{Bmatrix} -1/2 \\ -3/2\end{Bmatrix}. && (76)
\end{aligned}
$$

In the spin case, we have multiple possibilities for the constituent gluon (spin 1) to couple to the three quark system (spin 1/2) to give an overall spin 1/2 $qqqg$-state. $< S^3 \cdot S^g >_{MA}$ is easy to evaluate because the diquark is in an antisymmetric state, i.e., it has spin 0. Thus the gluon and the third quark carry the total spin of the hadron, 1/2.

$$
\begin{aligned}
< S^3 \cdot S^g >_{MA} &= \frac{1}{2}\Big[(S^3+S^g)^2-(S^3)^2-(S^g)^2\Big] && (77)\\
&= \frac{1}{2}\left[\begin{Bmatrix} C_S^{3g}\{\mathbf{4}\} \\ C_S^{3g}\{\mathbf{2}\}\end{Bmatrix} - C_S^q\{\mathbf{2}\} - C_S^g\{\mathbf{3}\}\right] && (78)\\
&= \frac{1}{2}\left[\begin{Bmatrix} 15/4 \\ 3/4\end{Bmatrix} - \frac{3}{4} - \frac{8}{4}\right] = \begin{Bmatrix} 1/2 \\ -1\end{Bmatrix}. && (79)
\end{aligned}
$$

The determination of the $< S^3 \cdot S^g >_{MS}$ value is more delicate. The diquark is in a spin 1 state, so the gluon and the third quark can be either in an $S = 1/2$ or $S = 3/2$ state to couple with the diquark to an overall $qqqg$ having spin 1/2. We have to rewrite the $qqqg$-state in terms of gluon-third quark states in order to evaluate the expectation value in eqn. 77, namely we have to see how often $|gq>_{S=1/2}$ and $|gq>_{S=3/2}$ are involved. This

is done with the help of Clebsch-Gordan coefficients. We use the notation $|l, m>_X$, where l labels the spin representation, m is the projection on the z-axis and X indicates which particles form that state. All Clebsch-Gordan coefficients are taken from the Particle Data Group book [1]. First we couple the gluon state $|1>_g$ to the mixed symmetric part of the three quark state $|\frac{1}{2}>_{qqq}$, to get a doublet.

$$|\tfrac{1}{2},\tfrac{1}{2}>_{qqqg} \;=\; \sqrt{\tfrac{2}{3}}|1,1>_g|\tfrac{1}{2},-\tfrac{1}{2}>_{qqq} - \sqrt{\tfrac{1}{3}}|1,0>_g|\tfrac{1}{2},\tfrac{1}{2}>_{qqq} \tag{80}$$

In this formula we substitute the mixed symmetric $|\frac{1}{2}>_{qqq}$ states,

$$\begin{aligned} |\tfrac{1}{2},\tfrac{1}{2}>_{qqq} &= \sqrt{\tfrac{2}{3}}|1,1>_{12}|\tfrac{1}{2},-\tfrac{1}{2}>_3 - \sqrt{\tfrac{1}{3}}|1,0>_{12}|\tfrac{1}{2},\tfrac{1}{2}>_3 \\ |\tfrac{1}{2},-\tfrac{1}{2}>_{qqq} &= \sqrt{\tfrac{1}{3}}|1,0>_{12}|\tfrac{1}{2},-\tfrac{1}{2}>_3 - \sqrt{\tfrac{2}{3}}|1,-1>_{12}|\tfrac{1}{2},\tfrac{1}{2}>_3, \end{aligned} \tag{81}$$

and get

$$\begin{aligned} &|\tfrac{1}{2},\tfrac{1}{2}>_{qqqg}= \\ &\quad\sqrt{\tfrac{2}{3}}\sqrt{\tfrac{1}{3}}|1,1>_g|1,0>_{12}|\tfrac{1}{2},-\tfrac{1}{2}>_3 - \sqrt{\tfrac{2}{3}}\sqrt{\tfrac{2}{3}}|1,1>_g|1,-1>_{12}|\tfrac{1}{2},\tfrac{1}{2}>_3 \\ &\quad-\sqrt{\tfrac{1}{3}}\sqrt{\tfrac{2}{3}}|1,0>_g|1,1>_{12}|\tfrac{1}{2},-\tfrac{1}{2}>_3 + \sqrt{\tfrac{1}{3}}\sqrt{\tfrac{1}{3}}|1,0>_g|1,0>_{12}|\tfrac{1}{2},\tfrac{1}{2}>_3 . \end{aligned} \tag{82}$$

We have to rewrite this in terms of $|l, m>_{3g}$-states with definite values.

$$\begin{aligned} |\tfrac{3}{2},\tfrac{3}{2}>_{3g} &= |1,1>_g|\tfrac{1}{2},\tfrac{1}{2}>_3 \\ |\tfrac{3}{2},\tfrac{1}{2}>_{3g} &= \sqrt{\tfrac{1}{3}}|1,1>_g|\tfrac{1}{2},-\tfrac{1}{2}>_3 + \sqrt{\tfrac{2}{3}}|1,0>_g|\tfrac{1}{2},\tfrac{1}{2}>_3 \\ |\tfrac{3}{2},-\tfrac{1}{2}>_{3g} &= \sqrt{\tfrac{2}{3}}|1,0>_g|\tfrac{1}{2},-\tfrac{1}{2}>_3 + \sqrt{\tfrac{1}{3}}|1,-1>_g|\tfrac{1}{2},\tfrac{1}{2}>_3 \\ |\tfrac{3}{2},-\tfrac{3}{2}>_{3g} &= |1,-1>_g|\tfrac{1}{2},-\tfrac{1}{2}>_3 \\ |\tfrac{1}{2},\tfrac{1}{2}>_{3g} &= \sqrt{\tfrac{2}{3}}|1,1>_g|\tfrac{1}{2},-\tfrac{1}{2}>_3 - \sqrt{\tfrac{1}{3}}|1,0>_g|\tfrac{1}{2},\tfrac{1}{2}>_3 \\ |\tfrac{1}{2},-\tfrac{1}{2}>_{3g} &= \sqrt{\tfrac{1}{3}}|1,0>_g|\tfrac{1}{2},-\tfrac{1}{2}>_3 - \sqrt{\tfrac{2}{3}}|1,-1>_g|\tfrac{1}{2},\tfrac{1}{2}>_3 \end{aligned} \tag{83}$$

Inverting these equations we find:

$$\begin{aligned} |1,1>_g|\tfrac{1}{2},-\tfrac{1}{2}>_3 &= \sqrt{\tfrac{1}{3}}|\tfrac{3}{2},\tfrac{1}{2}>_{3g} + \sqrt{\tfrac{2}{3}}|\tfrac{1}{2},\tfrac{1}{2}>_{3g} . \\ |1,0>_g|\tfrac{1}{2},-\tfrac{1}{2}>_3 &= \sqrt{\tfrac{2}{3}}|\tfrac{3}{2},-\tfrac{1}{2}>_{3g} + \sqrt{\tfrac{1}{3}}|\tfrac{1}{2},-\tfrac{1}{2}>_{3g} . \\ |1,0>_g|\tfrac{1}{2},\tfrac{1}{2}>_3 &= \sqrt{\tfrac{2}{3}}|\tfrac{3}{2},\tfrac{1}{2}>_{3g} - \tfrac{1}{\sqrt{3}}|\tfrac{1}{2},\tfrac{1}{2}>_{3g} . \end{aligned} \tag{84}$$

We substitute these results in eqn. 82 and find:

$$\begin{aligned} |\tfrac{1}{2},\tfrac{1}{2}>_{qqqg} = &-\tfrac{2}{3}|1,-1>_{12}|\tfrac{3}{2},\tfrac{3}{2}>_{3g} \\ &+\tfrac{2}{3}\sqrt{\tfrac{2}{3}}|1,0>_{12}|\tfrac{3}{2},\tfrac{1}{2}>_{3g} \\ &-\tfrac{2}{3\sqrt{3}}|1,1>_{12}|\tfrac{3}{2},-\tfrac{1}{2}>_{3g} \\ &+\tfrac{1}{3\sqrt{3}}|1,0>_{12}|\tfrac{1}{2},\tfrac{1}{2}>_{3g} \\ &-\tfrac{\sqrt{2}}{3\sqrt{3}}|1,1>_{12}|\tfrac{1}{2},-\tfrac{1}{2}>_{3g} \end{aligned} \tag{85}$$

From this follows

$$< S^3 \cdot S^g >_{MS} \;=\; \frac{12+8+4}{27} < S^3 \cdot S^g >_{MS}^{S=3/2} + \frac{1+2}{27} < S^3 \cdot S^g >_{MS}^{S=1/2} \tag{86}$$

$$= \frac{24}{27}(\frac{1}{2}) + \frac{3}{27}(-1) = \frac{9}{27} = \frac{1}{3}. \tag{87}$$

With these values, $<\sum_i S^i\cdot S^g F^i\cdot F^g>$in the total spin 1/2 state is

$$
\begin{aligned}
&<\sum_i S^i\cdot S^g F^i\cdot F^g>= \\
&\quad =\frac{3}{2}\left\{<S^3\cdot S^g>_{MA}<F^3\cdot F^g>_{MA}+<S^3\cdot S^g>_{MS}<F^3\cdot F^g>_{MS}\right\} && (88)\\
&\quad =\frac{3}{2}\left\{(-1)(-\frac{3}{2})+(\frac{1}{3})(-\frac{1}{2})\right\}=\frac{3}{2}\left\{\frac{9}{6}-\frac{1}{6}\right\} && (89)\\
&\quad =2. && (90)
\end{aligned}
$$